M_s , with the c.g. from the midchord as x_s . The half-aircraft fuselage is modeled as an equivalent mass at a distance of x_f from the root midchord. Further, both planform area and aspect ratio were kept as 1.0 so that results for other cases could be extrapolated easily. The material is assumed to be standard aluminum, with properties in SI units. Results are obtained for a symmetric flight maneuver for the stability derivatives $C_{L\alpha}$, $C_{m\alpha}$, C_{Lq} , and C_{mq} (lift and pitching moment coefficient derivatives with respect to angle of attack α and pitch rate q), which are calculated at a flight dynamic pressure of $10^4 \ N/M^2$, significantly lower than the divergence value. Further, although a Mach number M of 0.2 is selected, higher values in the subsonic zone will only mean higher flexible derivatives because of a correspondingly higher load, which could be extrapolated from corresponding rigid load ratios.

The results for flexible derivatives, including the divergence dynamic pressure Q_{div} , are obtained for the parameters Λ , c_r/c_r , x_f/c_r , M_s/M_f , and x_s/c_r , as given in Tables 1-5. Here, the variations in Λ and c_r/c_r are determined such that total wing sur-

Table 1 Sensitivity of stability derivatives to aircraft c.g. shift^a

x_f/c_r	$C_{I_{lpha}}$	C_{m_k}	C_{Lq}	C_{mq}	Qaiv
-0.20	7.132	0.965	4.711	-0.068	0.309×10^{5}
-0.10	6.072	1.300	4.432	0.237	0.517×10^{5}
0.00	4.895	1.453	3.800	0.404	0.352×10^{7}
0.10	4.862	1.861	3.541	0.618	0.957×10^{5}
0.20	5.803	2.717	3.804	1.036	0.426×10^{5}

 $^{{}^{}a}M_{s}/M_{f} = 0.05$, $\Lambda = 0$ deg, $c_{t}/c_{r} = 1.0$, $x_{s}/c_{t} = 0.0$, M = 0.2, $Q = 10^{4}$.

Table 2 Sensitivity of stability derivatives to wing taper ratio^a

c_t/c_r	$C_{I_{lpha}}$	C_{m_k}	C_{Lq}	C_{mq}	$Q_{ m div}$
1.000	4.895	1.453	3.800	0.404	0.352×10^{7}
0.800	4.958	1.206	4.412	0.258	0.163×10^{7}
0.600	5.019	0.881	4.560	0.018	0.157×10^{7}
0.400	5.059	0.443	5.065	-0.384	0.169×10^{7}
0.200	5.119	-0.171	5.746	-1.087	0.188×10^{7}

 $^{{}^{}a}M_{s}/M_{f} = 0.05$, $\Lambda = 0$ deg, $x_{f}/c_{r} = 1.0$, $x_{s}/c_{t} = 0.0$, M = 0.2, $Q = 10^{4}$.

Table 3 Sensitivity of stability derivatives to midchord sweep^a

Λ	$C_{I\alpha}$	C_{m}	C_{Lq}	C_{mq}	$Q_{ m div}$
-20.0	7.733	3.476	3.058	0.632	0.241×10^{5}
-10.0	5.343	2.009	3.135	0.437	0.554×10^{5}
0.00	4.895	1.453	3.800	0.404	0.352×10^{7}
10.0	7.848	1.696	5.827	0.553	0.269×10^{5}
20.0	10.710	1.423	7.650	0.311	0.184×10^{5}

 $^{{}^{}a}M_{s}/M_{f} = 0.05, x_{f}/c_{r} = 0.0, c_{t}/c_{r} = 1.0, x_{s}/c_{t} = 0.0, M = 0.2, Q = 10^{4}.$

Table 4 Sensitivity of stability derivatives to tip-store mass^a

M_s/M_f	$C_{I_{\alpha}}$	C_{m_k}	C_{Lq}	C_{mq}	$Q_{ m div}$
0.00	4.920	1.447	3.819	0.399	0.224×10^{6}
0.01	4.915	1.448	3.815	0.400	0.231×10^{6}
0.02	4.910	1.450	3.811	0.401	0.239×10^{6}
0.03	4.905	1.451	3.807	0.402	0.251×10^{6}
0.04	4.900	1.452	3.804	0.403	0.312×10^{7}
0.05	4.895	1.453	3.800	0.404	0.352×10^{7}

 $a_{c_t/c_r} = 1.0, \Lambda = 0 \text{ deg}, x_t/c_r = 0.0, x_s/c_t = 0.0, M = 0.2, Q = 10^4.$

Table 5 Sensitivity of stability derivatives to tip-store c.g. shift^a

x_s/c_t	$C_{I_{lpha}}$	C_{m_k}	C_{Lq}	C_{mq}	$Q_{ m div}$
-0.20	4.837	1.411	3.498	0.282	0.929×10^{5}
-0.10	4.735	1.400	3.605	0.332	0.138×10^{6}
0.00	4.895	1.453	3.800	0.404	0.352×10^{7}
0.10	5.418	1.608	4.121	0.511	0.126×10^{6}
0.20	6.321	1.885	4.582	0.661	0.468×10^{5}

 $[\]frac{1}{a}c_{t}/c_{r} = 1.0, \Lambda = 0 \text{ deg}, x_{f}/c_{r} = 0.0, M_{s}/M_{f} = 0.05, M = 0.2, Q = 10^{4}.$

face area remains constant. Table 1 shows that both $C_{L\alpha}$ and C_{Lq} reduce with an increase in x_f/c_r (+ve for aft movement), while $C_{m\alpha}$ and C_{mq} increase. In contrast, Table 2 shows that both $C_{L\alpha}$ and C_{Lq} increase, while $C_{m\alpha}$ and C_{mq} decrease as taper increases. The results in case of the midchord sweep Λ , presented in Table 3, show that sweepback has a greater influence on the effective aerodynamic coefficients than sweep forward, and that $C_{m\alpha}$ and C_{mq} decrease with an increase in Λ . Table 4 presents the effect of M_s/M_f variation when the store c.g. is on the elastic axis: all quantities are nearly invariant and only the divergence speed reduces noticeably. This also is in conformity with basic free aircraft effect. Finally, the effect of x_s/c_t , shown in Table 5, is the reverse of that observed in Table 1. These results for stability derivatives make it clear that free aircraft flexible force and moment coefficients can be substantially different even with small changes in geometric and mass configurations. Further, it is seen that desirable characteristics for free aircraft are possible by suitably adjusting the various geometry and inertia parameters, making the present study a useful de-

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Sensitivity of Aeroelastic Efficiencies of Subsonic Delta Wings to Partial Root Support

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Introduction

ODERN military aircraft employ wings with a delta ✓ shape, which provides good aerodynamic and aeroelastic performance. Delta wings are generally constructed using a single spar as the wing torsion box, with a width of about 60-85% of the total chord; thus there is only a partial connection of the wing root to the fuselage. Because the fuselage is much stiffer than the wing, it is possible to model the wing junction with the fuselage as a clamp for structural analysis of the wing. Further, during the preliminary design stage, the position of this torsion box with respect to the center chord is a variable based on many considerations. A study has investigated the sensitivity of the static aeroelastic characteristics of a generic, large-aspect-ratio, forward-swept wing to shape parameters. Another study examined the effect of spar size and location on the bending-torsion modal coupling of small-aspect-ratio, swept, and tapered plates.² It is well known that the location and size of the torsion box have a strong influence on the elastic deformation pattern of the wing and, therefore, have the potential to significantly alter the overall aerodynamic lift and pitching moment efficiencies from static aeroelastic corrections. Information about the sensitivity of these efficiencies

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Fig. 1 Geometry of a generic delta wing planform with partial root clamp.

Fig. 2 Variation of aeroelastic efficiencies with dynamic pressure for a) \bar{d} and b) \bar{x}_s values for 45-deg delta planform at Mach 0.2.

to the size and location of the torsion box is considered very useful for making minor adjustments that enable desirable aerodynamic characteristics. This Note demonstrates the sensitivity of aeroelastic efficiencies of a generic equivalent plate delta wing for different values of the leading-edge sweep and root-clamp configuration, and for two values of the subsonic flight Mach number.

Solution Procedure

The general aeroelastic problem of partially clamped cantilever plates cannot be solved exactly and needs a suitable numerical solution procedure for both the aerodynamic and structural calculations. A general-purpose static aeroelastic analysis software, STAAC (Ref. 3), has recently been developed that is based on the assumed-modes approach described in the work of Nicot and Petiau. The software uses the finite element formulation for structural analysis and the doublet-lattice formulation for aerodynamic analysis. Validation of the STAAC result is done for a simply supported, rectangular, semi-infinite plate in pure torsion, for which the exact analytical solution has been provided by Fung. A comparison of divergence dynamic pressure for this case indicates that STAAC overpredicts

it by about 5%. This is because the analytical solution⁵ is obtained for pure beam boundary conditions, whereas STAAC uses four-noded plate elements. The preceding validation also shows that a 15×15 discretization for both aerodynamic and structural calculations gives converged results. Thus, STAAC is considered to be an acceptable analytical tool for the aeroelastic efficiencies of a partially supported delta wing.

Example, Numerical Results, and Discussion

Figure 1 shows the geometry and dimensions of a generic delta wing as a plate of uniform thickness t, root chord c_r , and semispan b. The partial root clamp is characterized by its non-dimensional width \bar{d} (= d/c_r) and nondimensional offset from root chord center \bar{x}_s (= x_s/c_r). In practice, parameter d can be taken as the width of the wing torsion box at the root and x_s can be taken as the location of torsion box center with respect to the center of root chord, which can be estimated easily for a given delta planform. Two cases of leading-edge sweep, 45 and 60 deg, are considered for analysis under the constraint that the planform area is constant at $1.0M^2$, which provides normalized results suitable for overall wing geometry optimization. The plate material is taken as standard aluminum, with properties in SI units, for the purpose of analysis.

Numerical results for a symmetric flight maneuver for the lift efficiency η_L and the pitching moment efficiency η_M are obtained for two subsonic Mach numbers (0.2 and 0.85), two values of \bar{d} (1.0 and 0.6), and two values of \bar{x}_s (= ± 0.067 ; + for shift toward leading edge) to provide sensitivity behavior in the subsonic range. Figures 2-4 present the results for the efficiencies as a function of flight dynamic pressure Q, whose range is determined consistent with the Mach numbers selected. A further validation of the STAAC results for the delta wing problem has been done by comparing the rigid lift curve slope $C_L\alpha_r$ values mentioned in Figs. 2-4 with Data Compendium (DATCOM) (Ref. 5) results; STAAC values are found to be within 7% of the corresponding DATCOM values, thus establishing their reasonable accuracy.

Fig. 3 Variation of aeroelastic efficiencies with dynamic pressure for a) \bar{d} and b) \bar{x}_s values for 45-deg delta planform at Mach 0.85.

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Role of Modal Interchange on the Flutter of Laminated Composite Wings

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Introduction

THE vibration and flutter behavior of laminated composite wings (with cantilever end conditions) has been investigated by a number of research workers. 1-6 In a recent article it was shown that the flutter behavior of composite wings can be very different from their metallic counterparts and is mainly influenced by ply orientation and sweep angle. Bending-torsion coupling arising from ply orientations in a laminate plays a very important role and is a major factor influencing the flutter speed.

Investigators examining the flutter behavior of composite wings often observed some unexpected blips or abrupt changes occurring at certain fiber angles of the laminate, e.g., Figs. 7 and 10 of Ref. 1, and Figs. 11 and 12 of Ref. 4. These observations were confirmed by the present authors, 7 who concluded that the primary cause for these blips lies in the modal contributions at flutter, arising from ply orientations in the laminate. In this Note, further investigations are reported by looking into this unusual feature and pinpointing its underlying cause.

For illustrative purposes, one of the example wings of Ref. 7 that exhibited the characteristics mentioned in the preceding paragraph is further studied. First, a modal elimination technique is used to establish the number of dominant normal modes that contributed to the flutter behavior for different ply angles in the laminate. Next, the flutter mode is computed using selective normal modes that were found to be primarily responsible to cause flutter. Finally, contributions from each normal mode to the flutter mode are isolated in each case, and their relative individual contributions are studied. The results are discussed and some conclusions are drawn.

Method of Analysis

The method of analysis is essentially that of Ref. 7. However, some salient features of the theory are briefly summarized as follows.

Fig. 4 Variation of aeroelastic efficiencies with dynamic pressure for a) \bar{d} and b) \bar{x}_s values for 60-deg delta planform at Mach 0.85.

It is seen from Fig. 2a that both η_L and η_M decrease with a decrease in \bar{d} and an increase in dynamic pressure Q (in SI units), indicating that the overall c.p. for $\bar{x}_s = 0$ is behind the effective elastic axis. A comparison of results in Figs. 2a and 3a shows that the general trend for different \bar{d} values is the same for both a low subsonic Mach of 0.2 and a high subsonic Mach of 0.85, and that the only difference for these two Mach numbers is in terms of the actual values of both η_L and η_M which are higher for 0.85M. This is clearly due to the higher value of loading itself for this Mach 0.85, and therefore it can be concluded that results for 0.2M correctly capture the behavior of the aeroelastic efficiencies for the entire subsonic range. Figures 2b and 3b present the effect of \bar{x}_s on η_L and η_M . Once again, the trends are very similar, except for actual magnitudes of efficiencies. In addition, Figs. 2b and 3b indicate that η_L changes direction for $\bar{x}_s = -0.067$ (support movement toward trailing edge), causing the effective elastic axis to move closer to or slightly behind the effective c.p. at a particular value of Q. This may be because divergence can occur at a lower dynamic pressure for -ve values of \bar{x}_s .

This causes reversal of efficiencies for Q greater than its divergence, and also causes a lower value of Q for higher Mach numbers because wing loads are higher at higher Mach numbers. Results in Figs. 3a and 4a present the comparison of η_L and η_M for two different Λ of 45 deg and 60 deg, for the same Mach number of 0.85. Higher efficiencies for the 60-deg sweep are a result of lower structural deformations, which result from a lower aspect ratio of 2.3 vs 4.0 for the 45-deg sweep. However, the overall trend with the dynamic pressure for the 60-deg sweep is similar to those observed for the 45deg sweep at both low and high Mach values. This Note has clearly demonstrated that aeroelastic efficiencies of delta wings in the subsonic regime are fairly sensitive to parameters \bar{d} and \bar{x}_s , and that these sensitivities can help a designer in suitably configuring the structural parameters of a delta wing box to achieve the desired efficiency.

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